

Unquenched Numerical Stochastic Perturbation Theory

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The inclusion of fermionic loops contribution in Numerical Stochastic Perturbation Theory (NSPT) has a nice feature: it does not cost so much (provided only that an FFT can be implemented in a fairly efficient way). Focusing on Lattice $SU(3)$, we report on the performance of the current implementation of the algorithm and the status of first computations undertaken.

1. Introduction

At Lattice 2000 we discussed how to include fermionic loops contributions in Numerical Stochastic Perturbation Theory for Lattice $SU(3)$, an algorithm which we will refer to as UNSPT (Unquenched NSPT). Our main message here is that unquenching NSPT results in not such a heavy computational overhead, provided only that an *FFT* can be implemented in a fairly efficient way. *FFT* is the main ingredient in constructing the fermion propagator by inverting the Dirac kernel order by order. For a discussion of the foundations of UNSPT we refer the reader to [1].

2. Lattice $SU(3)$ UNSPT on APEmille

The need for an efficient *FFT* is what forced us to wait for APEmille: our *FFT* implementation mimic [2], which is based on a $1 - \dim FFT$ plus transpositions, an operation which asks for local addressing on a parallel architecture. UNSPT has been implemented both in single and in double precision, the former being remarkably robust for applications like Wilson loops. To estimate the computational overhead of unquenching NSPT one can inspect Table 1. We report execution times of a fixed amount of sweeps both for quenched and unquenched NSPT. On both columns the growth of computational time is consistent with the the fact that every operation is performed order by order. On each row the

growth due to unquenching is roughly consistent with a factor $5/3$. One then wants to understand the dependence on the volume, which is the critical one, the propagator being the inverse of a matrix: this is exactly the growth which has to be tamed by the *FFT*. One should compare execution times at a given order on $L = 8$ and $L = 16$ lattice sizes. Note that $L = 8$ is simulated on an APEmille board (8 FPUs), while $L = 16$ on an APEmille unit (32 FPUs). By taking this into account one easily understands that *FFT* is doing its job: the simulation time goes as the volume also for UNSPT (a result which is trivial for quenched NSPT). Notice that at this level one has only compared crude execution times: a careful inspection of autocorrelations is anyway not going to jeopardize the picture. As for the dependence on N_f (number of flavours), it is a parametric one: one plugs in various numbers and then proceed to fit the polynomial (in N_f) which is fixed by the order of the computation. It is then reassuring to find the quick response to a change in N_f which one can inspect in Figure 1 (which is the signal for second order of the plaquette at a given value of the hopping parameter K).

3. Benchmark computation I: Wilson loops

We now proceed to discuss some benchmark computations. A typical one is given by Wilson loops. In Figure 2 one can inspect the first five

Table 1

Execution times of a fixed number of sweeps for quenched and unquenched NSPT (see main text for details).

lattice size - order - APEmille resource	$N_f = 0$	$N_f \neq 0$
8 - g^6 - board	29	50
8 - g^8 - board	51	85
8 - g^{10} - board	82	134
16 - g^6 - unit	-	212

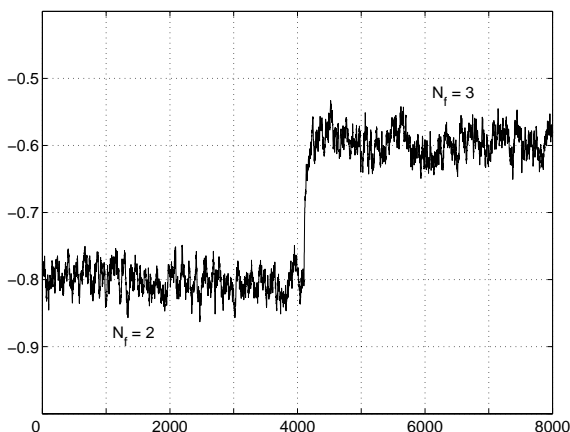


Figure 1. The effect of changing the number of flavours (on the fly).

orders¹ of the basic plaquette at a given value of hopping parameter K , for which analytic results can be found in [3]: going even higher in order would be trivial at this stage². Apart for being an easy benchmark, we are interested in Wilson loops for two reasons. First of all we are completing the unquenched computation of the Lattice Heavy Quark Effective Theory Residual Mass (see [4] for the quenched result). On top of that we also keep an eye on the issue of whether one can explain in term of renormalons the growth of

¹All our expansions are written in powers of β^{-1} .

²Notice anyway that these computations are performed at a given value of hopping parameter K , but with no mass counterterm (see later).

the coefficients of the plaquette. There is a debate going on about that (see [5]), the other group involved having also started to make use of NSPT. In the renormalon framework the effect of N_f can be easily inferred from the β -function, eventually resulting in turning the series to oscillating signs.

4. Benchmark computation II: the Wilson fermions Critical Mass

In Figure 3 we show the signal for one loop order of the Critical Mass for Wilson fermions (two loop results are available from [6]). The computation is performed in the way which is the most standard in Perturbation Theory, *i.e.* by inspecting the pole in the propagator at zero momentum. This is already a tough computation. It is a zero mode, an *IR* mass-cutoff is needed and the volume extrapolation is not trivial. On top of that one should keep in mind that also gauge fixing is requested. The coefficients which are known analytically can be reproduced. Still one would like to change strategy in order to go to higher orders (which is a prerequisite of all other high order computations). The reason is clear: we have actually been measuring the propagator S , while the physical information is actually coded in $\Gamma_2 = S^{-1}$ (one needs to invert the series and huge cancellations are on their way). Notice anyway that the fact that the Critical Mass is already known to two-loop makes many interesting computations already feasible.

5. Conclusions

Benchmark computations in UNSPT look promising, since the computational overhead of

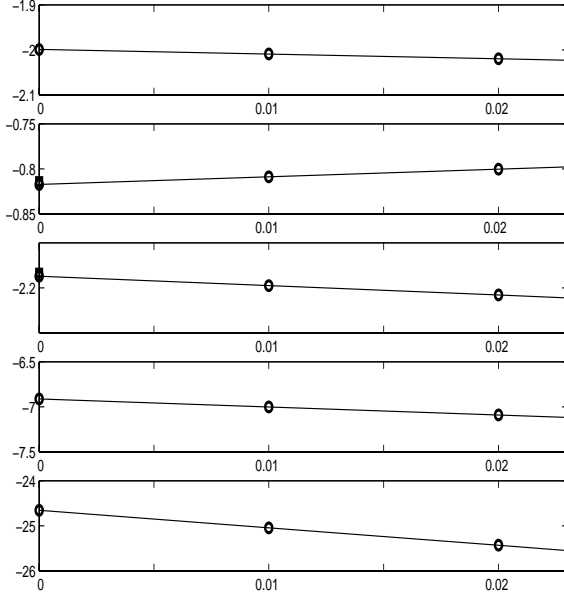


Figure 2. First five orders of the basic plaquette for $K = 0.16$ and $N_f = 2$ on a $L = 8$ lattice. On x -axes different values of the time step τ used in integrating Langevin equation: the result has to be extrapolated to $\tau = 0$. Analytic results for second and third order (first one is independent of sea fermions) in infinite volume are marked with a different symbol and they are hardly distinguishable.

including fermionic loops contributions is not so huge. This is to be contrasted with the heavy computational effort requested for non perturbative unquenched lattice QCD. This in turn suggests the strategy of going back to perturbation theory for the (unquenched) computation of quantities like improvement coefficients and renormalisation constants. The Critical Mass being already known to two loops, many of these computations are already feasible at α^2 order. We have only discussed the implementation of the algorithm on the APemille architecture. We can also rely on a C^{++} implementation for PC's (clus-

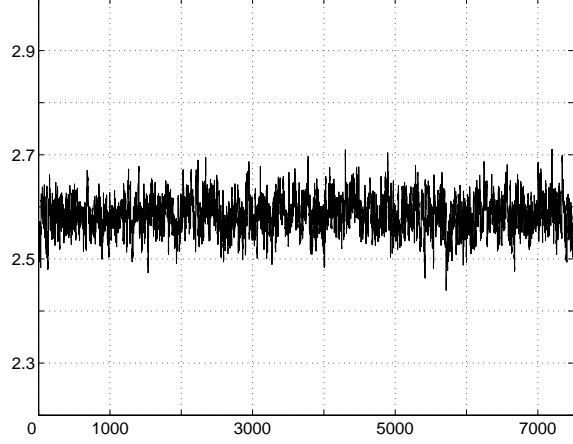


Figure 3. The signal for the first order of the Critical Mass on a $L = 8$ lattice with a cutoff mass of $m' = 0.01$, at a given value of the time step used in the integration of Langevin equation. Analytic result on the same volume and with the same cutoff is $2.557\dots$

ters) which is now at the final stage of development.

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